



FIGURE 6. A slightly smoother flat torus, generated by Vincent Borrelli, Saïd Jabrane, Francis Lazarus and Boris Thibert.

the letters "unny" together with a "B" lying on top of them more closely approximate the tile, as shown in the lower left of fig. 7). The bunny is topologically a sphere, so its Euler characteristic is two. With this, the fact that each tile (face) has four edges and four vertices, and knowing that there are 72 faces, it's possible to work out that each vertex has, on average, slightly fewer than four faces meeting at it (in fact $4/37$ less than 4). So we would expect to see lots of vertices with four faces, and some with fewer than four. However, there is a vertex at the tip of each of the two ears with only one face next to it (corresponding to very positive curvature) and a few others with three faces. This is balanced out by a number of vertices with more than four faces (roughly corresponding to points with negative curvature). The pattern of the "Bunny" word gets mapped onto each tile in an approximately angle-preserving way – the size of the design varies, but it's mostly undistorted otherwise.

Oskar van Deventer is a prolific puzzle designer, a member of a community of designers using 3D printing. Many (but by no means all) of his puzzles are "twisty puzzles", inspired by the classic Rubik's cube. The cube has been generalized in an astonishing variety of ways: to all of the other regular polyhedra, many irregular polyhedra, with parts that rotate in many different ways, and so on. Van Deventer holds the world record for the largest physical Rubik's cube variant, with his *Over the top* $17 \times 17 \times 17$ cube, see fig. 8.

Twisty puzzles are connected to some of the topics we have looked at in this book: They are generally made out of polyhedra, and the way they can be twisted is closely related to symmetries of those polyhedra. But there is much more going on in their designs – they are feats of engineering. It isn't obvious how an ordinary $3 \times 3 \times 3$ Rubik's cube holds itself together as it turns, let alone something as complicated as Oskar's *Over the top* $17 \times 17 \times 17$.



FIGURE 7. "*Bunny*" *Bunny*, by Craig S. Kaplan and me. The pattern on the four-sided tiles is shown in the lower left.

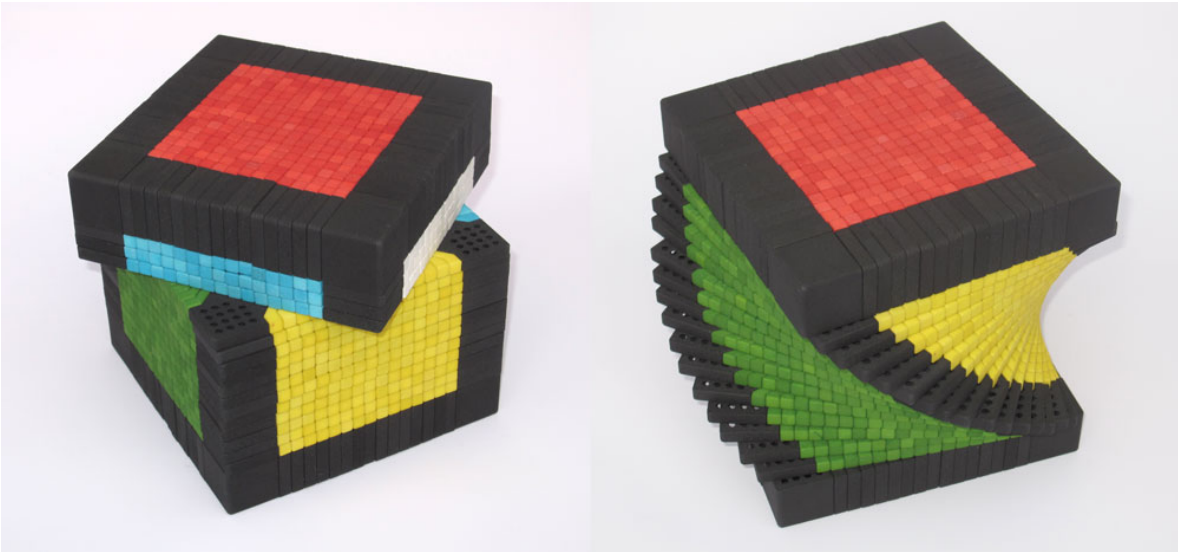


FIGURE 8. *Over the top* $17 \times 17 \times 17$, by Oskar van Deventer.

On the subject of surprising mechanisms, fig. 9 shows *Magic Gears*, also by Oskar van Deventer. Three strangely shaped gears fit onto axles mounted on a base. On the left, the gears work in the same way as ordinary gears: if you turn the first gear counterclockwise then the second turns clockwise, which makes the third turn counterclockwise. The directions that the gears turn alternate as you go along. On the right however, if you turn the middle gear upside down then paradoxically, all three gears turn in the *same* direction. (The spacing of the axles has to be slightly different as well.)

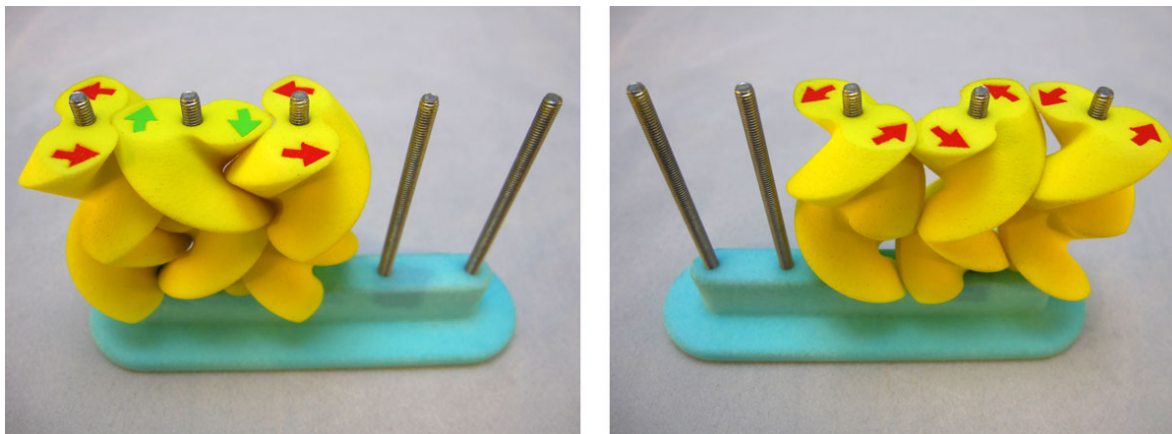


FIGURE 9. *Magic Gears* by Oskar van Deventer.

Finally, also on the subject of unusual gears, fig. 10 shows *Triple gear*, another strange mechanism. These three rings make the (3,3) torus link, as in fig. 5-7. As I mentioned in chapter 5, the Borromean rings have been used as a symbol of unity, because cutting any of the three loops makes the other two fall apart. This doesn't happen for the (3,3) torus link, but Triple gear is unified in a different way: No single gear can turn on its own, but all three rings can turn together.

I hope that you have enjoyed this brief tour of some 3D printable mathematics. There are many more beautiful 3D prints that I could have talked about, and many many more that I couldn't, because they haven't been designed yet. 3D printing technology and 3D design software are both getting ever more accessible, so if you want to get started on filling in the gaps, that would be great. Seriously though, 3D printing is finding its way into high school and college courses, and the Maker Movement is bringing these tools to everyone. Making a 3D printed model of a mathematical object is an excellent way to learn about it. By the time you're done you know it very well – you have to in order to be able to tell the computer and the printer what the shape is. So I encourage you to try designing your own. It's difficult for me to recommend a good resource for this, since everything is both very new and changing quickly, so I won't. But take a look around online, there are lots of free resources and programs out there.

Almost all of the 3D models used in this book are available to download from the website, 3dprintmath.com, and many of these may be good starting points for printing mathematical models on your own printer. Or maybe not – for most of the prints in